

Merged

One chain, two decisions

*The combined physics, parameters and objective for the Baghewala digital twin
— source for the deck, the simulator, and model training.*

PROBLEM STATEMENT

SIH26120 · Oil India Limited · Software · Smart Automation

USE FOR

Presentation content · Simulator · ML parameter ranges · Judge Q&A

OPEN ITEMS

Three, listed in section six

Revised 12 September 2026 after a document review. Oil SG and the Walther constant A are now stated exactly (equation 3). The float-limit timing is corrected (equation 5). Open item 1 is rewritten: the viscosity curves agree at reservoir temperature and spread most at steam temperature, not the other way round. Missing constants are added (equation 1). New tier-D assumptions: the cooling-constant law (equation 2), the production-rate law and pump displacement (equation 4), the injection and soak phases (equation 7) and the measured signals (equation 8). The cumulative-oil definition is corrected. Steam ranges are narrowed to Baghewala field data. Reservoir pressure is flagged as uncertain. The pump-intake equation is corrected. Symbol clashes are removed. An unsupported error figure is removed.

Revised 24 September 2026. Pump speed is now marked as derived, not measured. Decision D10 sets the realistic operating range at 0.9–3.4 SPM. The old 2–12 SPM range and the 12 SPM mechanical ceiling stay in the document for the worked example only. They are our assumption (tier D), not a device specification and not a running speed. Stroke length is split by pump unit type:

conventional 1–3 m, hydraulic 3–6 m, 1–6 m overall. A new parameter, pump unit type, is added to section five. Section seven is new. It gives the derivation, dated evidence for each source, a note on a figure not to use, and the current Oil India field numbers, read live. Equation 5 now uses the peak rod speed, set by pump unit type, with two new tier-D assumptions. This addresses part of open item 2. The worked example in section four is unchanged.

One

The problem

Baghewala, Rajasthan. Heavy crude at 14–19° API, roughly 11,500 cP at 50 °C (the midpoint of the reported 10,000–13,000 cP), in a reservoir with a native temperature of 46–50 °C. At that thickness it will not flow, and reservoir pressure is too low to push it anyway. Two interventions are mandatory: steam to thin the oil, and a sucker rod pump to lift it.

Steam heats the near-wellbore rock. Over the following weeks that heat drains into the surrounding cold formation, viscosity climbs back, and two things degrade at once — less oil reaches the well, and the rod string can no longer fall through the thickening oil on its downstroke. It goes slack, gets shock-loaded on every stroke, and fatigues until it fractures.

Both effects have one cause: reservoir temperature. Steam scheduling and pump operation are currently managed by separate teams on historical practice, so nothing connects them.

That disconnect is the problem statement. The deliverable is a well-to-surface digital twin that observes the current state, predicts how it evolves, and prescribes both decisions from the same prediction.

Two

The causal chain

Steam volume + quality → heated zone V_h → temperature $T(t)$ → viscosity μ

then splits

↓ mobility → productivity J_o → inflow q_{in} → oil rate

↑ rod damping → maximum safe pump speed N_{max}

both converge on

SOR — barrels of steam per barrel of oil

The split is the key structural point. Production potential and mechanical safety are not two problems that happen to share a well. They are two consequences of one variable, which is why optimising them separately is guaranteed to be suboptimal.

Heated zone volume matters beyond its obvious role: a larger heated zone has proportionally less surface exposed to cold rock, so it cools more slowly. Equation 2 makes this exact — the cooling constant follows the zone's volume-to-surface ratio. Steam volume therefore buys *duration*, not just temperature. This is the physical variable through which "inject more now" versus "wait" becomes computable rather than habitual.

Three

The objective

Maximise oil produced per unit of steam injected, subject to pump speed staying below the rod-floating limit.

Stated formally, over a cycle:

OBJECTIVE FUNCTION

$$\min \text{ SOR} = V_{\text{steam}} / N_p \quad \text{subject to} \quad N(t) \leq N_{\text{max}}(\mu(t))$$

Decision variables: steam volume V_s , soak time t_{soak} , re-steam date t_{cut} , and pump speed $N(t)$ each day.

N_p = cumulative oil over the cycle, $N_p = \int \min(q_{\text{in}}, Q_{\text{pump}}) dt$, where Q_{pump} is the pump displacement at the set speed (equation 4). Do not multiply q_{in} by fillage: low fillage is the result of low inflow, so that counts the same limit twice. SOR uses total N_p , which includes the oil the well would give cold.

Two consequences worth stating explicitly in the deck.

Rod safety is a constraint, not the goal. If avoiding failures were the objective, the optimum would be to run at minimum speed forever — no failures, almost no oil. Failures matter because they halt production and incur workover cost. The value of knowing N_{max} precisely is that it lets the pump run *close* to the limit rather than conservatively far below it.

Re-steam timing moves SOR more than pump speed does. Pump speed is a daily optimisation within a cycle. Re-steam timing is the cycle-level decision, and it directly sets the numerator of SOR. Too early wastes steam on a well still producing; too late means weeks of crawling output at 4 SPM.

Four Equations

1 · STEAM → HEATED ZONE [ASSUMPTION — REDUCED-ORDER ENERGY BALANCE]

$$V_h = \eta \cdot m_s \cdot [x \cdot L + c_p(T_s - T_i)] / [M_R(T_s - T_i)]$$

Steam enthalpy divided by rock heat requirement gives heated rock volume. Radius $r_h = \sqrt{(V_h / \pi h)}$.

Constants: $m_s = \rho_w \cdot V_s$ with $\rho_w = 1,000 \text{ kg/m}^3$, because V_s is a cold-water-equivalent volume. $L = 1,716.6 \text{ kJ/kg}$, the latent heat of steam at 250 °C. $c_p = 4.187 \text{ kJ/(kg} \cdot \text{K)}$. $M_R = 2,350 \text{ kJ/(m}^3 \cdot \text{K)}$. $T_i = 50 \text{ °C}$. $h = 9.92 \text{ m}$ net heated thickness (tier D: back-solved from the worked radius; replace with field net pay).

$T_s = 250 \text{ °C}$ is an effective downhole value (tier D). Field surface steam is 280–305 °C; the downhole temperature after wellbore losses is not reported. L is held at its 250 °C value.

Worked: 2,000 m³ CWE, $x = 0.70$, $T_s = 250 \text{ °C}$, $T_i = 50 \text{ °C}$ gives 2,039 kJ/kg. With $\eta = 0.5$: $V_h = 4,338 \text{ m}^3$, $r_h = 11.8 \text{ m}$.

Feeds: equation 2, through the heated radius r_h that sets the cooling constant τ ; and equation 4, through r_h in the two-zone rate law.

2 · THERMAL BALANCE → TEMPERATURE [PHYSICS; COOLING LAW IS AN ASSUMPTION]

$$C_{\text{eff}} \cdot dT_r/dt = \dot{Q}_{\text{steam}} - \dot{Q}_{\text{loss}} - \dot{Q}_{\text{prod}}$$

General form, valid during injection, soak and production. For the production phase with steam off and loss proportional to the temperature gap (heat carried out by the produced fluid is folded into the loss term), this integrates to a closed form usable directly in a dashboard:

$$T(t) = T_i + (T_s - T_i) \cdot e^{-t/\tau} \quad \text{where } \tau = C_{\text{eff}}/U_{\text{eff}}$$

Cooling-constant law [assumption, tier D]: τ is proportional to the heated zone's volume-to-surface ratio. For a cylinder of radius r_h and height h that ratio is $r_h \cdot h / 2(r_h + h)$, so

$$\tau = \tau_{\infty} \cdot r_h / (r_h + h), \quad \tau_{\infty} = 30 \cdot (r_{h,\text{ref}} + h) / r_{h,\text{ref}} = 55.2 \text{ days}$$

$r_{h,\text{ref}} = 11.80 \text{ m}$ is the worked radius, so $\tau = 30$ days exactly in the worked case. τ rises with steam volume and levels off towards 55 days; there is no hard cap. Over the simulator range (500–4,000 m³) τ runs from 21 to 35 days.

Worked ($\tau = 30 \text{ d}$; t is the production day, and day 0 is the first day after soak): day 0 → 250 °C · day 30 → 124 °C · day 60 → 77 °C · day 90 → 60 °C.

3 · TEMPERATURE → VISCOSITY [PHYSICS — ASTM D341 / WALTHER, ONE CONSTANT CALIBRATED]

$$\log_{10}(\log_{10}(v + 0.7)) = A - B \cdot \log_{10}(T)$$

T in kelvin, v in cSt, μ (cP) = $v \times \text{SG}$ with **SG = 0.963** (from $\rho_o = 963 \text{ kg/m}^3$; about 15.4 °API, inside the field range). Anchored on 11,500 cP at 50 °C — the midpoint of the reported 10,000–13,000 cP, not a single measurement — with B = 3.5.

A is computed from the anchor and not rounded: $A = \log_{10}(\log_{10}(11,500/0.963 + 0.7)) + 3.5 \cdot \log_{10}(323.15) = 9.3933$. Rounding A to 9.394 gives 11,684 cP at the anchor, 1.6% high.

Worked: day 0 → 5 cP · day 30 → 93 cP · day 60 → 1,148 cP · day 90 → 4,465 cP · fully cold (50 °C) → 11,500 cP

Do not substitute a simple exponential. Oil viscosity changes too steeply with temperature for one exponential to hold from 50 °C to 250 °C. The double-logarithm form (ASTM D341) is the standard for petroleum liquids.

4 · VISCOSITY → PRODUCTION [PHYSICS; RATE LAW AND PUMP CONSTANTS ARE ASSUMPTIONS]

$$J_o \propto k \cdot k_{ro} / \mu_o \quad \cdot \quad q_{in} = J_o (P_r - P_{wf})$$

Thin oil flows more readily, so the productivity index rises as viscosity falls. Inflow is productivity times drawdown.

Rate law [assumption, tier D — Boberg–Lantz two-zone radial flow]: the heated zone (r_w to r_h , viscosity $\mu_h(t)$) and the cold zone (r_h to r_e , viscosity μ_c) resist flow in series. Drawdown is held constant in v1.

$$q_{in}(t) = q_{cold} \cdot \ln(r_e/r_w) / [(\mu_h(t)/\mu_c) \cdot \ln(r_h/r_w) + \ln(r_e/r_h)]$$

$r_w = 0.1$ m, $r_e = 100$ m, $q_{cold} = 10$ bbl/d (the unstimulated rate per well), $\mu_c = \mu(T_i) = 11,500$ cP, $\mu_h(t)$ from equation 3 at $T(t)$. When the heated oil is thin, the cold zone still limits flow, so the rate has a natural ceiling of $q_{cold} \cdot \ln(r_e/r_w) / \ln(r_e/r_h) = 3.23 \times q_{cold}$ at $r_h = 11.8$ m. As the zone cools the rate falls back to q_{cold} , not to zero. q_{cold} is set so the worked case lands inside the reported SOR band.

Pump displacement [assumption, tier D]: $Q_{pump} = A_p \cdot S \cdot N \cdot 1440 \cdot \eta_v$ in m^3/d (N in strokes per minute, 1,440 minutes per day). Plunger 1.75" ($A_p = 1.552 \times 10^{-3} m^2$), volumetric efficiency $\eta_v = 0.8$, pump setting depth $D_p = 1,100$ m. v1 uses the polished-rod stroke S for the plunger stroke.

Surface rate: $q = \min(q_{in}, Q_{pump})$ at the set speed N . Slowing the pump costs oil only when Q_{pump} falls below q_{in} .

Pump fillage [assumption, tier D], 24 September 2026: $F_{fill} = \min(1, q_{in} / Q_{pump})$. It is the share of the pump's stroke volume that the reservoir fills. It is defined on the displacement Q_{pump} above, which uses the polished-rod stroke and includes η_v . It is not defined on the card's shorter plunger stroke. The card (equation 6) fills this share of each stroke with liquid; η_v stands for plunger slippage, which the card's chamber model leaves out. Below 0.85 the card shows fluid pound. This is the one fluid-pound threshold in the model. **Worked** (production day 0): $q_{in} = 32.3$ bbl/d and $Q_{pump} = 168.6$ bbl/d, so $F_{fill} = 0.19$. At $N = 6$ SPM the pump is much larger than the inflow, so every worked day is below 0.85.

Worked ($2,000 m^3$, $x = 0.70$, $S = 2.5$ m, $N = 6$ SPM): $Q_{pump} = 168.6$ bbl/d, so the reservoir limits the rate all cycle. $q = 32.3$ bbl/d on day 0 · 31.7 on day 30 · 26.4 on day 60 · 17.3 on day 90 · 12.7 on day 120. Over 120 production days (trapezoidal rule, 1-day steps) $N_p = 2,946$ bbl. Steam = $2,000/0.159 = 12,579$ bbl, so **SOR = 4.27**.

Coupled to the pump by: pump intake $P_{intake} = P_{casing} + \rho_o \cdot g \cdot h_{above}$ (casing-head pressure plus the oil column in the annulus above the pump), with $P_{wf} \approx P_{intake}$. Pump discharge $P_{discharge} = P_{wh} + \rho_o \cdot g \cdot D_p + \Delta P_f$ (tubing side; friction adds when flow is upward). v1 does not compute these; it holds drawdown constant (open item 3).

Limitation: linear productivity index assumed; non-linear Vogel-type inflow at high drawdown is out of scope for v1.

5 · VISCOSITY → PUMP SPEED LIMIT [ASSUMPTION — STATIC FORCE BALANCE, OURS, NEEDS CALIBRATION]

$$N_{\max} = \min(30 \cdot v_f / S, 12 \text{ SPM}) \quad \text{where} \quad v_f = (\rho_s - \rho_o) \cdot g \cdot d_r^2 \cdot \ln(d_t / d_r) / (8\mu)$$

Rod weight against viscous drag gives the fall speed available; the pump allows a half-stroke period of 30/N seconds to complete it. If required exceeds available, the string goes slack. μ is in Pa·s here (1 cP = 0.001 Pa·s). With the table values, $N_{\max} = 128,240 / (\mu_{\text{cP}} \cdot S)$. The 12 SPM mechanical ceiling is tier D (the top of the N range).

Note, 24 September 2026: 12 SPM is our assumed ceiling for the worked example (tier D). It is not a device specification, and Baghewala wells do not run near it. Section seven derives the realistic range from the field's reported output: 0.9–3.4 SPM.

Peak rod speed, 24 September 2026:

$$N_{\max} = \min(30 \cdot v_f / (f \cdot S), N_{\text{mech}})$$

f is the ratio of peak to mean downstroke rod speed. The rod must fall at the **peak** speed, not the mean. $f = 1$ reproduces the static balance as first written; the worked example keeps it. A conventional unit's sinusoidal motion gives $f = \pi/2$.

[Assumption, tier D] A hydraulic unit accelerates, cruises and decelerates in each half stroke. Let r be the share of the period the rod spends speeding up, or slowing down, at each end of a half stroke; a full reversal at the top or bottom takes $2r$. The rod covers the stroke S in $(\frac{1}{2} - r) \cdot T$ at its peak speed, so $v_{\text{peak}} = S / ((\frac{1}{2} - r) \cdot T)$, while the mean downstroke speed is $S / (\frac{1}{2} \cdot T)$. So $f = \frac{1}{2} / (\frac{1}{2} - r)$. With $r = 0.08$ (assumed; HRP and SLB describe near-constant travel but publish no reversal time), $f = 0.5 / 0.42 = 1.19$.

[Assumption, tier D] The oil around the rods is taken at reservoir temperature. Oil India states Baghewala uses vacuum insulated tubing and thermal wellheads (Rajasthan Fields page, read 24 September 2026). These keep the oil in the tubing near reservoir temperature. Colder oil in the upper tubing would lower $N_{\max}$ further.

Why it matters: in fully cold oil (11,500 cP at 50 °C), a conventional unit at its 3 m maximum stroke has $N_{\max} = 128,240 / (11,500 \times \pi/2 \times 3) = 2.37 \text{ SPM}$. The constant 128,240 is for the default 7/8 in rod in 2 7/8 in tubing. 2.37 SPM is inside the 0.9–3.4 SPM field range, so the rods can float. With $f = 1$ the limit is 3.72 SPM, above the range, and the model could never show a conventional well floating.

Worked ($S = 2.5 \text{ m}$): day 0–60 → no binding limit · day 90 → **11.5 SPM** · fully cold → **4.5 SPM**

Key finding for the deck: the limit does not bite for about the first three months of a cycle. In the worked case it first falls below 12 SPM on day 88 (60.4 °C, 4,275 cP); across strokes of 1–3 m that day ranges from 83 to 169. Then it collapses rapidly. That end-of-cycle window is where the mechanical damage occurs, and it is precisely the period nobody currently monitors.

Floating risk [assumption, tier D]: risk = $N / N_{\max}$. Above 1, the set speed exceeds the safe speed and the rods float.

THE TWIN'S DAILY RECOMMENDATION [RULE, TIER D — SPEC §3.8, 25 SEPTEMBER 2026]

Each production day, the twin proposes a pump speed and a stroke. It also proposes the next steam date. It uses equations 1–5 with the well's fixed details and one set of well numbers: the textbook values, or a Model 1 estimate. It never sees the true state. The operator decides. Before production (injection and soak) the twin proposes nothing.

1 · Speed for a target fillage [tier D]:

$$N_{\text{fill}} = q_{\text{in}} / (0.85 \times Q_{\text{pump}} \text{ per SPM})$$

Q_{pump} per SPM is equation 4 at $N = 1$ SPM. At N_{fill} the pump fills to 0.85, the fluid-pound threshold of the card (equation 6). A faster pump runs part-empty and pounds. A slower pump leaves oil in the reservoir.

2 · Drive range [tier D, D10]:

N_{fill} is clamped to the drive range 0.9–3.4 SPM (section seven).

3 · The safety cap, applied last and always [tier D]:

$$N \leq 0.9 \times N_{\text{max,est}}$$

$N_{\text{max,est}}$ is equation 5 at the twin's estimated viscosity and the stroke under test. The result is rounded **down** to the 0.1 SPM grid, so rounding never moves it above the cap.

When the cap is below 0.9 SPM, the recommendation is 0 SPM: stop the pump. No drive speed is then safe. The floor must not win over the cap, because that would recommend a speed above 90 % of the estimated safe speed. The operator is warned (floating) and decides. This replaces spec §3.8 step 4, which gave 0.9 SPM with a warning.

4 · Stroke search [tier D]:

Steps 1–3 run for each stroke on a 0.1 m grid inside the well's stroke range. The oil rate at each stroke is $q = \min(q_{\text{in}}, Q_{\text{pump}})$ (equation 4). The twin picks the stroke with the most oil. A tie goes to the lowest rod speed $S \times N$: fewer reversals and less drag.

5 · Stroke change needs 10 % more oil [tier D]:

On a conventional unit, a stroke change means moving the crank pin. That needs a crew. So the twin keeps the current stroke, with its own best speed, unless another stroke gives more than 10 % more oil.

Warnings [tier D]:

Floating: the floating risk at the current speed (equation 5) is above 0.9, or the cap is below the floor. Fluid pound: the fillage at the recommended controls is below 0.85. This happens when the inflow is so low that even 0.9 SPM empties the pump.

6 · Next steam date [tier D]:

$$d^* = \operatorname{argmax}_{d \geq 30} Q_{\text{cum}}(d) / (t_{\text{shut-in}} + d + 1)$$

The twin forecasts the rate for production days 0–365 at the recommended controls. $Q_{\text{cum}}(d)$ is the cumulative oil to production day d , by the trapezoid rule of the day stepper. $t_{\text{shut-in}}$ is the whole days of injection and soak. The denominator counts calendar days: shut-in days plus production days 0 to d . The twin proposes re-steaming the day after d^* . This maximises oil per calendar day for the cycle. At that day the marginal rate has fallen to the cycle's calendar-average rate. d^* is never before production day 30. The earliest day wins a tie. The rule follows bestCutoff in Pratham's `impact-analysis.ts` (spec §17), rewritten on whole field days.

THE OPERATOR'S PAGE [RULES, TIER D — PHASE 4 PLAN, 25 SEPTEMBER 2026]

The operator's page shows one well: the twin's state, the recommendation, warnings, steam and the dynamometer card. The rules below add no physics. They read the twin (the recommendation above and equations 4–6) and the well's readings. They never read the true state. The code is `src/lib/physics/operator.ts`.

1 · Control guard [tier D, 25 September 2026]:

A pump setting is valid when the speed is 0 (the pump is stopped) or 0.9–3.4 SPM on the 0.1 SPM grid (section seven, D10), and the stroke is inside the well's stroke range (its fixed details). Any other setting is refused. The day stepper refuses a speed above 3.4 SPM, so a refused setting never reaches the simulator. One function checks the form and the server.

2 · Safe speed now [tier D, 25 September 2026]:

$$N_{\text{safe}} = \lfloor \min(\text{cap}, 3.4) / 0.1 \rfloor \times 0.1 \quad \text{if cap} \geq 0.9; \quad \text{else } 0$$

$\text{cap} = 0.9 \times N_{\text{max,est}}$ at the current stroke (step 3 above). The speed is rounded **down** to the grid, with the same rule as the recommendation, so it is never above the cap. When the cap is below 0.9 SPM, the pump stops. The one-click button only lowers the speed: it never sets a speed above the current one. The stroke does not change.

3 · Warnings [tier D, 25 September 2026]:

Both warnings use the twin's state today at the **current** controls. Floating: the floating risk $N / N_{\text{max,est}}$ is above 0.9, or today's recommendation found the cap below the floor and the pump still runs. Fluid pound: the pump fillage is below 0.85 (equation 6). A stopped or shut-in pump has no fillage, so it gets neither warning. The one-click safe speed shows when the floating warning is on.

4 · Cycle and instantaneous SOR [tier D, 25 September 2026]:

$$\text{SOR}_{\text{cycle}} = V_{\text{steam}} / Q_{\text{cycle}} \quad \text{SOR}_{\text{now}} = (V_{\text{steam}} / t_{\text{cycle}}) / q_{\text{today}}$$

V_{steam} is the cycle's steam in barrels of cold-water equivalent ($\text{m}^3 \div 0.159$). Q_{cycle} is the cycle's measured oil to today, in bbl. t_{cycle} is the calendar days of the cycle to today, today included (injection, soak and production). q_{today} is today's measured rate. Each is "—" when its oil or rate is 0.

Identity. $\text{SOR}_{\text{now}} / \text{SOR}_{\text{cycle}} = (Q_{\text{cycle}} / t_{\text{cycle}}) / q_{\text{today}}$. So SOR_{now} rises above $\text{SOR}_{\text{cycle}}$ exactly when today's rate falls below the cycle's calendar-average rate. That is the re-steam rule of step 6 above: re-steam when the marginal rate falls to the calendar-average rate. The page shows both numbers; the operator decides.

5 · Card gap [tier D, 25 September 2026]:

$$\text{gap} = \text{mean}_i |F_{\text{surface},i} - F_{\text{expected},i}| / (\max F_{\text{expected}} - \min F_{\text{expected}})$$

F_{surface} is the measured surface card. F_{expected} is the card the twin expects today with no fault (equation 6 at the twin's viscosity and fillage). Both have 100 points at the same speed, stroke and time steps, so point i is the same instant of the stroke. The gap is a share of the expected load range. The page warns when the gap is at or above `CARD_GAP_WARN_SHARE` (0.35, from the study below). The gap says only that the card differs from the twin. It names no fault: that is Model 2's job (D12).

Study: the warning threshold [tier D, 25 September 2026]:

`scripts/study-card-gap.ts` sets the threshold. It does not tune it. Cases: the 81 number sets of the twin check (low, textbook and high for each of the 4 well numbers), both unit types, speeds 0.9, 2.0 and 3.4 SPM, and production days 10, 60 and 120, at the unit's default stroke and the worked steam job. That is 1,458 no-fault cases. The measured card is equation 6 at the case's numbers, 100 rod nodes, with the simulator's card noise. The expected card is the twin's card at the textbook numbers, at the same speed, stroke and production day: the twin before Model 1, so the gap includes the twin's own error. The full grid ran, with no reduction (857 s).

No-fault cases	n	p50	p90	p95	p99	max
All	1,458	0.055	0.162	0.211	0.313	0.401
Conventional	729	0.049	0.142	0.190	0.240	0.264
Hydraulic	729	0.060	0.182	0.238	0.347	0.401

CARD_GAP_WARN_SHARE = the 99th percentile of all no-fault gaps (nearest rank, 0.313), rounded up to 0.05: **0.35**.
0.4 % of the no-fault cases are at or above it. The same cases with each injected fault at severity 0.5:

Fault (severity 0.5)	p10	p50	p90	At or above 0.35
Gas interference	0.076	0.110	0.243	0.5 %
Leaking travelling valve	0.054	0.107	0.159	0.2 %
Leaking standing valve	0.051	0.099	0.196	0.8 %
Parted rod	0.476	0.874	1.068	100.0 %
Unseated pump	0.274	0.453	0.542	77.8 %

Limit. Fewer than half of three fault classes pass the threshold: gas interference, the leaking travelling valve and the leaking standing valve (under 1 % each). Their gaps sit inside the spread that the twin's own error (textbook numbers against the well's numbers) and the card noise give a normal pump. So the gap warns reliably of a parted rod and mostly of an unseated pump. It does not warn of gas or a leaking valve at severity 0.5. No gap warning does not mean no fault. Naming those faults needs Model 2 (D12). The threshold stays at 0.35; a lower one would warn on normal pumps.

6 · ROD DYNAMICS → DYNAMOMETER CARD [PHYSICS — GIBBS DAMPED WAVE EQUATION; PUMP AND FAULT MODELS ARE ASSUMPTIONS, TIER D]

$$\partial^2 u / \partial t^2 = a^2 \cdot \partial^2 u / \partial x^2 - c \cdot \partial u / \partial t \quad \text{where} \quad a = \sqrt{(E/\rho_s)}$$

Updated 24 September 2026. v1 did not compute this equation. It is now computed. It gives the dynamometer card: the load on the polished rod against its position over one stroke.

x runs down the rod string, from the surface (x = 0) to the pump (x = D_p). u(x, t) is the rod displacement, up positive, measured from the rod hanging still under its own weight. So gravity and buoyancy drop out of the equation. The tension at depth x is W(x) – EA·∂u/∂x, where W(x) is the buoyant weight of the rod below x. E = 2.05 × 10¹¹ Pa (carbon steel, std), so a = 5,110 m/s and EA = 7.94 × 10⁷ N for the 7/8" rod.

Damping — the link to viscosity:

$$c = 2\pi \cdot \mu / (\ln(d_t/d_r) \cdot \rho_s \cdot A_r) + \pi \cdot a \cdot v / (2L)$$

The first term is the viscous drag of equation 8, per unit length of rod and per unit rod mass. It uses **that day's oil viscosity** μ from equation 3. So the card changes shape as the well cools. The second term is Gibbs (1963) structural damping, with v = 0.05 (tier D; Gibbs uses 0.05–0.1) and L the rod length. **Worked:** c = 0.37 s⁻¹ on day 0 (4.8 cP), 2.68 s⁻¹ on day 60 (1,148 cP), 23.5 s⁻¹ fully cold (11,500 cP).

Numerical method: explicit central differences on 100 rod segments (Δx = D_p/100 = 11 m). The time step obeys the CFL condition Δt ≤ Δx/a with a safety factor of 0.9. It is rounded down so that a stroke holds a whole number of steps: Δt = 1.94 ms. The rod starts at rest. The model runs four strokes and records the fourth. The card is 100 points, evenly spaced in time, starting at the bottom of the stroke.

Top boundary — the carrier bar: the carrier bar moves the rod top with the polished-rod motion (equation 5: a sine wave for a conventional unit, constant speed with ramps for a hydraulic unit). The surface load is W_{rb} – EA·∂u/∂x at x = 0.

Rod floating — the carrier-separation rule: the carrier bar can pull the rod but cannot push it. If the surface load would fall below 0, the rod top leaves the carrier. The top is then free: its tension is 0, so EA·∂u/∂x = W_{rb} at x = 0. The rod falls under its own weight against the oil drag. When it falls back onto the carrier, it rejoins it. That re-contact is the shock the rods take. The card is marked "rod floating" when the rod leaves the carrier at any time in the recorded stroke. For a stiff rod, the surface load on the downstroke is W_{rb} minus the drag. So the rod leaves the carrier when the peak rod speed passes the fall speed v_f; this is equation 5. The card adds the rod's stretch, its inertia and the structural damping.

Bottom boundary — the pump: –EA·∂u/∂x = F at x = D_p, where F is the pump force (tension positive):

$$F = (P_{\text{discharge}} - P_{\text{chamber}}) \cdot A_p$$

The chamber is the space between the standing valve (SV) at the bottom and the travelling valve (TV) in the plunger. P_{intake} = ρ_o·g·h_{sub}, with h_{sub} = 100 m of oil above the pump intake (tier D): 0.94 MPa. P_{discharge} = P_{intake} + ρ_o·g·D_p = 11.34 MPa. The fluid load is (P_{discharge} – P_{intake})·A_p = ρ_o·g·D_p·A_p = 16.1 kN, the same as in equation 8.

1. Liquid does not compress. When the chamber is full of liquid and both valves are shut, the plunger cannot move. The rod holds it with any force from 0 to the fluid load. So the rod stretches to pick up the load at the bottom of the stroke, and contracts to release it at the top.
2. The plunger moves up only when the rod pulls harder than the fluid load. The SV then opens, and liquid enters at P_{intake}.
3. The plunger moves down only when the rod would push it. The TV then opens, and F = 0.
4. When the SV can admit no more liquid, the rest of the chamber is a vapour void at P_{chamber} ≈ 0 (vapour pressure neglected, tier D). On the downstroke the plunger falls through the void with the full load, then hits liquid: fluid pound.
5. Free gas follows P·V = constant (isothermal). Gas must be compressed to P_{discharge} before the TV opens. So gas gives a gradual load drop, not a sharp one.

6. The dead space (the chamber volume with the plunger at the bottom) is 5 % of the swept volume (tier D).

An explicit method cannot carry a very stiff spring at the bottom node. So each step solves the bottom node's move and the pump force together. F rises with the plunger's move, so there is one solution. It is found exactly for liquid and by bisection for gas.

Fillage [assumption, tier D]: in each upstroke the SV admits liquid up to $F_{\text{fill}} \times$ the plunger's swept volume. $F_{\text{fill}} = \min(1, q_{\text{in}} / Q_{\text{pump}})$ is the fillage of equation 4. The rest of the stroke draws the vapour void. η_v in Q_{pump} stands for plunger slippage; the chamber model leaves slippage out. A card is labelled fluid pound when $F_{\text{fill}} < 0.85$. This is the one fluid-pound threshold in the model (CARD_CONSTANTS.poundFillage).

Injected faults [tier D shape models]: each fault has a severity s from 0 to 1. The models give the textbook card shape. They are not calibrated to field cards.

Fault	Model
Gas interference / gas lock	A share $0.9 \cdot s$ of the swept volume enters as free gas at P_{intake} . The liquid share is $\min(F_{\text{fill}}, 1 - 0.9 \cdot s)$. After the TV opens, the gas that fits in the dead space at $P_{\text{discharge}}$ stays behind. It expands again on the next upstroke. $s = 1$ is gas lock.
Leaking travelling valve	The load the plunger keeps falls linearly with time through the upstroke. At the end of the upstroke it has lost $0.6 \cdot s$ of the fluid load.
Leaking standing valve	At the top of the stroke, liquid equal to $0.4 \cdot s$ of the swept volume returns to the annulus. The plunger then falls through a void before it releases the load. So the card looks like fluid pound: both lose liquid before the downstroke.
Parted rod	The rod parts at depth $D_p \cdot (1 - 0.8 \cdot s)$: between 20 % and 100 % of the depth. The rod below the part and the pump carry no load.
Unseated pump	The pump carries no fluid load. The whole rod still moves in the oil.

Label: when several faults apply, the card takes the first in this order: parted rod, unseated pump, gas interference, leaking travelling valve, leaking standing valve, rod floating, fluid pound, normal.

Peak-load tolerance: take the worked-case well on production day 60, with no fault and full fillage. The card's peak surface load must agree with equation 8's peak polished-rod load (50.2 kN) within 15 %. Equation 8 is quasi-static and uses the mean rod speed. The card is dynamic: its drag follows the true rod speed, and the load overshoots a little when the rod picks up the fluid load. So the two are not expected to agree exactly. If the card misses 15 %, the physics is checked; the tolerance is not widened. **Worked:** card peak 55.3 kN, 10 % above equation 8.

Cross-check with equation 5 (Stop 1, Q1): in fully cold oil (11,500 cP), the card is run at speeds 0.05 SPM apart. The first speed where the rod leaves the carrier is compared with equation 5's N_{max} . Neither model is tuned to the other. **Worked:** conventional unit, $S = 3$ m: card 2.40 SPM, equation 5 2.37 SPM, ratio 1.01. Hydraulic unit, $S = 5$ m: card 1.90 SPM, equation 5 1.87 SPM, ratio 1.01. The two agree within the 0.05 SPM step.

Limitation: in heavy oil the rod drag alone gives a wide surface card. For a rod of mass M the drag work per stroke is $c \cdot M \cdot \int v^2 dt$. On worked day 60 (1,148 cP) this gives a card area of 28 kN·m for an unseated pump and 18 kN·m for a rod parted at 60 % of the depth. The normal card is 64 kN·m. In thin oil (day 0, 4.8 cP) the same two faults give about 9 % of the normal area. The downhole card has no rod drag.

Cost: one card takes about 70 ms at 0.9 SPM, 31 ms at 2 SPM and 18 ms at 3.4 SPM in Node (npm run time:card). The number of steps grows with the stroke period.

Deferred: the Everitt–Jennings downhole inversion, which rebuilds the downhole card from a measured surface card. Equation 5 still sets N_{max} . The card is for fault classification, where the card *shape* carries the signal.

7 · CSS PHASES — INJECTION AND SOAK [ASSUMPTION, TIER D]

Production (equation 2) starts from a fixed state: the heated zone at T_s with volume V_h . Injection and soak must end in that state. They are modelled to be plausible, not precise.

Injection length: $t_{inj} = 1,000 \cdot V_s / (\dot{m}_s \cdot 24)$ days with $\dot{m}_s = 3,100$ kg/h (OIL, tier B). 2,000 m³ takes 26.9 days; the field practice of 14–21 days corresponds to 1,040–1,560 m³.

Injection temperature: $T(t) = T_i + (T_s - T_i)(1 - e^{-t/\tau_{inj}})$, $\tau_{inj} = 1$ day.

Heated volume during injection grows in proportion to injected heat: $V(t) = f_{inj} \cdot V_h \cdot t / t_{inj}$, with $f_{inj} = [c_p(T_s - T_i) + f_c \cdot x \cdot L] / [x \cdot L + c_p(T_s - T_i)]$ and $f_c = 0.5$. This is the sensible heat plus the latent heat of the half of the steam that has condensed by the end of injection. $f_{inj} = 0.705$ in the worked case.

Soak: T holds at T_s while the remaining steam condenses at saturation temperature, and V grows linearly from $f_{inj} \cdot V_h$ to V_h by the end of soak.

Production length: 120 days by default (t_{cut} range 60–365 days). **Days since steaming** = days since injection stopped (soak days plus production days).

v1 does not model the soak optimum. Soak time moves the calendar but does not change the production-start state.

8 · MEASURED SIGNALS [ASSUMPTION, TIER D]

A later stage infers the hidden state from signals a sensor can read. The forward model must therefore produce those signals.

Measured: wellhead temperature, pump load, flow rate q , set SPM and stroke, days since steaming. **Inferred:** reservoir temperature, viscosity, heated-zone size, floating risk.

Wellhead temperature (Ramey, 1962):

$$T_{wh} = T_{g,0} + g_G \cdot A - (T_{g,0} + g_G \cdot D + g_G \cdot A - T_r) \cdot e^{-D/A}$$

$g_G = (T_i - T_{g,0})/D$, surface temperature $T_{g,0} = 30$ °C, $D = 1,150$ m, relaxation length $A = a \cdot q$ with $a = 20$ m per bbl/d. Oil leaves the reservoir at T_r and cools towards the geotherm as it rises; a low rate cools more. Defined only while producing; no value during injection and soak.

Pump load — peak polished-rod load:

$$PPRL = W_{rb} \cdot (1 + S \cdot N^2/1790) + \rho_o \cdot g \cdot D_p \cdot A_p + 2\pi \cdot \mu \cdot v \cdot D_p / \ln(d_t/d_r)$$

$W_{rb} = (\rho_s - \rho_o) \cdot g \cdot (\pi d_r^2/4) \cdot D_p$ is the buoyant rod weight (28.8 kN). The second term is the fluid load on the plunger for a pumped-off well (16.1 kN). The third is viscous drag on the rods during the upstroke at the mean speed $v = S \cdot N/30$ (μ in Pa·s), using the same Couette drag as equation 5. $S \cdot N^2/1790$ is the Mills acceleration factor with S in metres.

Heated-zone size stays at V_h through production; only its temperature falls.

WORKED PRODUCTION CASE — ALL EQUATIONS

$V_s = 2,000 \text{ m}^3$, $x = 0.70$, $T_s = 250 \text{ }^\circ\text{C}$, $S = 2.5 \text{ m}$, $N = 6 \text{ SPM}$, all other values from the table in section five. Days are production days. Use this table to verify an implementation.

Note, 24 September 2026: $N = 6 \text{ SPM}$ sits inside the old 2–12 SPM model range, not the realistic 0.9–3.4 SPM range derived in section seven. The numbers below are kept exactly as computed. They stay the reference case for automated checks (`npm run verify:physics`).

Day	T (°C)	μ (cP)	$N_{\max}$ (SPM)	q (bbl/d)	T_{wh} (°C)	PPRL (kN)	Risk
0	250.0	4.8	12.00	32.3	73.0	46.4	0.50
30	123.6	93.1	12.00	31.7	51.3	46.7	0.50
60	77.1	1,148	12.00	26.4	41.2	50.2	0.50
90	60.0	4,465	11.49	17.3	36.2	61.4	0.52
120	53.7	7,998	6.41	12.7	34.4	73.2	0.94

Heated zone $4,338 \text{ m}^3$, $r_h = 11.8 \text{ m}$, $\tau = 30 \text{ d}$, injection 26.9 days. $N_p = 2,946 \text{ bbl}$, $\text{SOR} = 4.27$. At $N = 6 \text{ SPM}$ the rods start to float on production day 127; at $N = 8 \text{ SPM}$, on day 106.

Where ML attaches: Corrected = Physics + ML_residual, applied at the wellbore and dynamometer modules. Physics stays the backbone; models learn the discrepancy rather than replacing the mechanism. Two models fall out — one predicting the temperature decay curve for this well and cycle, one solving the constrained optimisation in section three.

Five

Parameters for model training

Confidence tiers — **A** stated in the problem statement, **B** published Oil India or SPE data, **D** our assumption, requires calibration. **std** marks physical constants and steam-table values.

Symbol	Meaning	Unit	Value / range	Conf.
$^\circ\text{API}$	Crude gravity	$^\circ\text{API}$	17–19 (PS) vs 14–17 (field); model uses SG $0.963 \approx 15.4$	A / B
T_i	Native reservoir temperature	$^\circ\text{C}$	46–48 (PS) vs ~50 BHT (field); model uses 50	A / B
μ_o	Viscosity at 50 $^\circ\text{C}$, at 50 s^{-1}	cP	10,000–13,000; model anchor 11,500 (midpoint)	B
D	Well depth, Jodhpur Sandstone	m	~1150	B
P_r	Reservoir pressure	ksc / MPa	~116 ksc = 11.4 MPa UNCERTAIN — CONFIRM	B
ϕ	Porosity	fraction	< 0.10 CONFIRM SOURCE — unusually low for this sandstone; no equation uses it	B
V_s	Steam volume per cycle	$\text{m}^3 \text{ CWE}$	Literature 300–12,000; Baghewala field 1,040–1,560 (3,100 kg/h for 14–21 days); simulator 500–4,000, default 2,000	B

Symbol	Meaning	Unit	Value / range	Conf.
$\dot{m}_s$	Steam injection rate	kg/h	~3,100	B
t_{inj}	Injection period	days	14–21 (field); model: $V_s/\dot{m}_s$	B
x	Steam quality at wellhead	fraction	0.60–0.80 (literature); 0.60–0.70 (field)	B
T_s	Steam temperature	°C	200–300 (literature); 280–305 surface (field); model 250 effective downhole	B / D
P_{inj}	Injection pressure	MPa	1–11 (≤ 1600 psig preferred); field steam 85–97 ksc (8.3–9.5 MPa)	B
t_{soak}	Soak time	days	3–14 (field: 50–60% of injection time)	B
t_{cut}	Production length before re-steam	days	60–365; model default 120	B / D
N	Pump speed	SPM	0.9–3.4 (D10, derived from field output — see section seven). The worked example keeps the old 2–12 model range, with 12 as its assumed ceiling	D
S	Stroke length	m	1–6 overall, by pump unit type: conventional 1–3, hydraulic 3–6	D
Unit type	Pump unit type	—	conventional or hydraulic; fixed per well, set at completion	D
SOR	Steam–oil ratio	—	3.0–5.2	B
RF	Recovery factor	%	<20 typical, up to 35	B
q_o	Field production rate (whole field)	bbl/day	Above 1,100, 56 wells drilled / 34 producing (Oil India Rajasthan Fields, read 24 Sep 2026); ~655 (July 2025) as history	B
ρ_w, c_p, L	Water density, specific heat, latent heat at 250 °C	kg/m ³ , kJ/(kg·K), kJ/kg	1,000 · 4.187 · 1,716.6	std
M_R	Rock volumetric heat capacity	J/(m ³ ·K)	2.35×10^6	D
η	Thermal efficiency	fraction	0.4–0.7; model 0.5	D
h	Net heated thickness	m	9.92 (back-solved from $r_h = 11.8$ m)	D
τ	Cooling time constant	days	$\tau_\infty \cdot r_h / (r_h + h)$, $\tau_\infty = 55.2$; 21–35 over the simulator range. τ_{ref} (τ at the worked radius) = 30; 20–40 is the range for per-well hidden numbers (brief, 24 September 2026)	D
τ_{inj}, f_c	Injection heating constant; steam condensed by end of injection	days, fraction	1 · 0.5	D
A, B	Walther constants	—	A = 9.3933 (computed from the anchor), B = 3.5	D
ρ_o / ρ_s	Oil / steel density	kg/m ³	963 / 7850 (SG 0.963)	D
d_r / d_t	Rod / tubing ID	m	0.0222 (7/8") / 0.062 (2 7/8")	D

Symbol	Meaning	Unit	Value / range	Conf.
r_w / r_e	Well radius / drainage radius	m	0.1 / 100	D
q_{cold}	Unstimulated (cold) rate per well	bbl/day	10; 8–12 is the range for per-well hidden numbers (brief, 24 September 2026)	D
A_p / η_v	Plunger area (1.75") / volumetric efficiency	m ² / fraction	1.552×10^{-3} / 0.8	D
D_p	Pump setting depth	m	1,100 (worked case); per well 900–1,140, at least 10 m above the reservoir top (Stop 1, Q5)	D
$T_{g,0} / a$	Surface temperature / Ramey relaxation constant	°C / m per bbl/d	30 / 20	D

Conversions — $1 \text{ ksc} = 98.07 \text{ kPa} \cdot 1 \text{ bbl} = 0.159 \text{ m}^3 \cdot 1 \text{ psi} = 6.895 \text{ kPa} \cdot 1 \text{ cP} = 0.001 \text{ Pa} \cdot \text{s} \cdot \nu(\text{cSt}) = \mu(\text{cP})/\text{SG} \cdot \text{SG} = 141.5/(131.5 + ^\circ\text{API})$ · the model uses SG = 0.963 throughout.

Pump unit type

Each well has one of two kinds of surface pump unit. It is set when the well is completed and does not change over the well's life.

A **conventional** unit is a beam pump — the familiar nodding donkey. A crank turns the beam, so the rod moves close to a sine wave: fast in the middle of the stroke, slow at each end. Stroke length 1–3 m.

A **hydraulic** unit moves the rod with a hydraulic cylinder. Speed stays near constant through most of the stroke, with a quick reversal at each end. Stroke length 3–6 m.

Both types are in use at Baghewala (Oil India, Rajasthan Fields page, read 24 September 2026 — quoted in section seven).

Fixed details of each well

Each well has fixed details. These are known engineering sizes. The superadmin enters them when the well is added. The rod must be one of the standard sizes below, because its coupling must fit the tubing. The plunger must be smaller than the tubing ID. The other sizes are listed for reference.

Item	Standard sizes	Source (read 24 September 2026)
Sucker rod diameter	$\frac{5}{8}$, $\frac{3}{4}$, $\frac{7}{8}$, 1 in; default $\frac{7}{8}$ in	API Spec 11B rod sizes; Octal Steel, "Sucker Rod Couplings", https://www.octalsteel.com/product/sucker-rod-couplings/
Rod coupling outside diameter (full size)	$\frac{5}{8}$ in rod: 1.500 in · $\frac{3}{4}$ in: 1.625 in · $\frac{7}{8}$ in: 1.812 in · 1 in: 2.187 in	API Spec 11B full-size couplings; Octal Steel, as above. https://www.sucker-rod.com/suckerrod/sucker-rod-couplings.html gives the same values, rounded to two decimals
Plunger (pump bore) diameter	$1\frac{1}{4}$, $1\frac{1}{2}$, $1\frac{3}{4}$, 2, $2\frac{1}{4}$ in (31.8, 38.1, 44.5, 50.8, 57.2 mm); default $1\frac{3}{4}$ in	API Spec 11AX, 11th edition, June 2001, basic bore sizes 125, 150, 175, 200, 225 (copy at https://pdfcoffee.com/api-spec-11ax-components-of-subsurface-pumps-pdf-pdf-free.html)
Tubing inside diameter	$2\frac{3}{8}$ in, 4.70 lb/ft: 1.995 in · $2\frac{7}{8}$ in, 6.50 lb/ft: 2.441 in · $3\frac{1}{2}$ in, 9.30 lb/ft: 2.992 in	API 5CT tubing; Quail Tools tubing data sheets, one for each size, https://www.quailtools.com/assets/images/specsheets/tubing/

The model refuses a set of details that cannot be a real well:

1. The pump depth is outside 900–1,140 m.
2. The rod is not a standard size.
3. The rod coupling is not smaller than the tubing ID. The coupling is wider than the rod, so the coupling sets the clearance. A 1 in rod (coupling 2.187 in) does not fit $2\frac{3}{8}$ in tubing (ID 1.995 in). A $\frac{7}{8}$ in rod (coupling 1.812 in) fits.
4. The plunger is not smaller than the tubing ID.
5. The stroke range is outside the range of the unit type.

The worked case keeps its own sizes: rod 0.0222 m and tubing ID 0.062 m. These are the $\frac{7}{8}$ in rod and the $2\frac{7}{8}$ in tubing, within rounding.

State vector for the twin: $X = [T_r, V_h, P_r, \mu_o, \lambda_o, q_{in}]$ with control input $u = [V_s, x, t_{soak}, N, S]$ and uncertainty σ . Every parameter carries value, unit, source, calibration method and confidence.

On the conflicting values

API gravity and native temperature differ between the problem statement and published field data. Preserve both rather than silently reconciling — but note that the field figures come from Oil India's own technical publications and SPE papers written by the field's operators, while the problem statement is an outreach document. Model with the field values; cite the discrepancy openly if asked. The model uses $T_i = 50$ °C and $SG = 0.963$.

Six Open items

1 · Second viscosity measurement

One reported value exists: 10,000–13,000 cP at 50 °C, and the model uses the 11,500 cP midpoint. Fitting Walther requires two points, so $B = 3.5$ is assumed. Because the curve is anchored at 50 °C, every B gives the same viscosity there. The spread between $B = 3.2$ and $B = 3.8$ (the higher viscosity divided by the lower, minus one) is 0% at the anchor. It grows in both directions — 57% at 30 °C ($B = 3.2$ gives 96,700 cP, $B = 3.8$ gives 151,700 cP) and 78% at 250 °C. So the uncertainty is smallest near reservoir temperature, where the well spends most of its life, and largest at steam temperature.

Inside the cycle, measured the same way, the spread is 41% at 77 °C and 77% at 124 °C. B from 3.2 to 3.8 moves the day the float limit binds only from day 86 to day 91, and changes the worked SOR from 4.35 to 4.20. A second measurement, ideally near 100 °C, would fix B and test whether the Walther form holds for this crude.

2 · Calibrate the floating limit against failure history

Equation 5 is a static force balance. It ignores rod stretch, elastic wave effects, plunger fluid load and mechanical friction, so it reads optimistic — real $N_{\max}$ is lower. Two more reasons it read optimistic; the second (peak speed) is now addressed by unit type.

First, it uses the heated-zone viscosity for the oil around the rods, but oil cools as it rises up the tubing, so the upper rods sit in thicker oil. This part stays open. See the tubing-temperature note under equation 5.

Second, it compared the rod's fall speed with the mean downstroke speed $S \cdot N/30$, while for near-sinusoidal motion the peak speed is $\pi/2$ higher — on that basis the day-90 limit would be 7.3 SPM, not 11.5. Addressed 24 September 2026, by unit type: equation 5 now divides by the peak-to-mean factor f ($\pi/2$ for a conventional unit, 1.19 for a hydraulic unit). The worked example keeps $f = 1$.

Oil India states it holds rod failure and pump unsettling history. Fit one correction factor so predicted $N_{\max}$ aligns with speeds at which failures actually occurred, and present it as calibrated rather than first-principles.

3 · Reservoir pressure has no governing dynamic

P_r appears in every state vector and drives q_{in} directly, but no dP_r/dt relationship is defined. Either add a simple depletion and support term, or state explicitly that it is held constant or externally supplied for $v1$, with the reason. $v1$ answer: P_r is held constant, and the rate law in equation 4 holds drawdown constant, so P_r does not enter any worked number.

The reported value is also uncertain. 116 ksc at 1,150 m is close to hydrostatic (about 115 ksc), which does not fit the problem statement's "low reservoir pressure". Surface steam at 85–97 ksc could not enter a reservoir at 116 ksc. The figure may be the initial pressure. Confirm it with Oil India before using it; it is not corrected here.

4 · Data to ask Oil India for

The calibration path forward needs:

- a second viscosity point, for Walther B;
- SRP logs with kWh;
- failure dates;
- per-cycle production;
- dynamometer cards.

Unmodelled, worth naming to judges: the crude is non-Newtonian — viscosity is quoted at a shear rate of 50 s^{-1} , so thickness depends on shear as well as temperature. Since rod motion through the fluid *is* a shear event, pump speed influences viscosity, which influences the safe pump speed. A feedback loop inside equation 5. Acknowledge as a known refinement; do not build it until the main chain works.

Seven

Pump speed — evidence and derivation (D10)

Decision D10 sets the pump speed range at 0.9–3.4 SPM. This replaces the 2–12 SPM range used earlier in the model. The new range is **derived, not measured** — no source reports the SPM a Baghewala pump actually runs at. It is worked out from the field's reported oil output and the pump's own displacement formula, which is equation 4 of this document. The working is below, followed by the dated sources.

PUMP SPEED RANGE — DERIVATION [EVIDENCE, TIER D]

$$\text{bbl/day} = D_{\text{plunger}}^2 \cdot 0.1166 \cdot S \cdot N \cdot \eta_v$$

D_{plunger} and S in inches, N in SPM. This is the field-unit form of equation 4's pump-displacement law (source 6 below). Both forms give the same number, checked below.

Worked, per SPM: $D_{\text{plunger}} = 1.75 \text{ in}$, $S = 2.5 \text{ m} = 98.4 \text{ in}$, $\eta_v = 0.8$. $1.75^2 \times 0.1166 \times 98.4 \times 0.8 = \mathbf{28.1 \text{ bbl/day for one SPM}}$. Check against equation 4: its worked case gives $Q_{\text{pump}} = 168.6 \text{ bbl/d}$ at $N = 6 \text{ SPM}$; $168.6 / 6 = 28.1$, the same figure.

Baghewala output per well [source 5]: under 20 bbl/day, rising to over 80, then settling at 40–50 bbl/day.

Dividing each rate by 28.1 bbl/day per SPM gives the speed it needs:

20 bbl/day → 0.7 SPM · 80 bbl/day → 2.9 SPM · 40–50 bbl/day (settled) → 1.4–1.8 SPM. Baghewala's reported output therefore needs about **0.7–2.9 SPM**. D10 adopts **0.9–3.4 SPM** as the realistic range — close to this computed need, with some margin.

The old 12 SPM ceiling, at the same stroke and plunger, would move $12 \times 28.1 = \mathbf{337 \text{ bbl/day}}$ — about 4 to 17 times a well's reported output, and roughly ten times the settled 40–50 bbl/day rate. A Baghewala well does not need that speed.

This speed is derived, not measured. It depends on the assumed plunger size (1.75 in, tier D, equation 4) and on the stroke used in the arithmetic (2.5 m, the worked-example value). A different plunger or a different stroke — see the ranges by pump unit type in section five — changes the answer.

Evidence, each read on 24 September 2026:

1. **Oil India, Rajasthan Fields.** <https://www.oil-india.com/rajasthan-fields> — states wells are "produced using conventional as well as hydraulic SRP units," with "thermal wellhead, VIT(Vacuum Insulated Tubings)"; crude viscosity "in the range of 10,000-13,000 cP at 50 deg C"; and "avg. depth of 1150 m." Confirms the claim. Current well counts and production are quoted below, under "Oil India field numbers, read live."
2. **Oil India tender, Sigma Tenders, June 2024.** <https://sigmatenders.com/tender/eKW3kaFrdRvbnCUfkqGJX> — an Oil India Limited tender, opened 5 June 2024, for "Hiring Services of Sucker Rod Pumping Unit SRP Hydraulic Sucker Rod Pumping HSRP Surface Unit ... for a Period of 01 Year." Confirms Oil India hires hydraulic SRP units. The page does not name "Rajasthan Projects" as the location — that part of the claim is not confirmed by this page.
3. **SLB, hydraulic pumping units.** <https://www.slb.com/products-and-services/innovating-in-oil-and-gas/completions/artificial-lift/sucker-rod-pumps/surface-pumping-units/hydraulic-pumping-units> — a direct fetch returned only the page's navigation menu, not its body text; the page appears to load its content by script. A search-engine excerpt of this same page gives "cylinder lengths of up to 240 in" and "Long strokes enable these hydraulic pumping units to maintain production at slower pumping speeds." 240 in $\approx$ 6.1 m, matching the claimed range.
4. **HRP International, above-ground cylinders.** <https://www.hrpi.com/Above-Ground-Cylinders> — lists stroke lengths "36, 60, 86, 120, 240, 336 (inches)," confirming 60–336 in. States "near-fixed traveling speeds" and that "the long stroking cylinder allows rods to travel at lower speeds." Confirms the claim.
5. **SPE 215309, Baghewala electrical downhole heater (Oil India, 2023).** <https://onepetro.org/SPEAPOG/proceedings-abstract/23APOG/2-23APOG/535203> — a direct fetch returned HTTP 403 Forbidden; OnePetro blocked the request. The same abstract wording was found through search-engine excerpts of this exact page: API gravity "in the range of 14–17" and viscosity "ranging from 8000 to 15000 cP @ 50°C" (this differs from source 1's 10,000–13,000 cP — both are genuine Oil India figures, from different documents; they are not reconciled here); and production "increased from less than 20 bbl/day to more than 80 bbl/day and stabilised at around 40-50 bbl/day." Confirms the claim, with the viscosity difference noted above.
6. **Lornmac, rod pump displacement formula.** <https://www.lornmac.com/rodPump.html> — gives "BPD = (DIA X DIA x 0.1166 X STROKE X SPM)," with DIA and STROKE in inches. Confirms the claim. The page does not show volumetric efficiency inside this formula — it has a separate "Efficiency, EFF" input field — so this document applies $\eta_v = 0.8$ separately, the same way equation 4 does.

Do not use — "5 SPM, 54 in stroke"

A figure of 5 SPM with a 54 in stroke appears next to the Baghewala fishing-operation paper on ResearchGate. Do not use it here. That ResearchGate page lists several papers together, and the figure belongs to a different field, not Baghewala.

Oil India field numbers, read live — 24 September 2026

The Oil India Rajasthan Fields page states, in its own words: "56 nos. of wells have already been drilled out of which 34 nos. of wells are in production," and "total oil production is above 1100 barrels/day." These are whole-field numbers, not per-well.

This is the current figure, read 24 September 2026. The parameter table's q_0 row carries the same figure, and keeps the earlier count, "~655 (July 2025)" bbl/day, as dated history: production grew between the two readings.